Interference and Interferometry

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Interference is the term used to signify a large class of phenomena in light, and interometry is the technique of high precision measurement based on these phenomena. Ordinarily rays of light crossing the same point in different directions do not interfere with each other; each ray is propagated as though that alone were present. But there are certain interesting cases when they do interfere with each other; the interference may be destructive, as when they cancel each other's effect, or may be constructive, as when they reinforce each other. Interference is a consequence of light being propagated in the form of waves.

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YOUNG'S EXPERIMENT

The classic experiment in interference is the one performed by Thomas Young in 1802. A source of light SL (see figure 1) that is placed behind a narrow slit illumines two other slits P and Q which are parallel and very close to each other. At some distance away is a screen which receives the light from the two slits. On the screen is seen a series of bright and dark fringes. If either of the two slits is covered the fringes disappear, and the screen is almost uniformly illumined. The combined effect due to the two slits is that at certain points there is no light at all, and at other points the brightness is four times that due to a single slit.

This puzzling phenomenon of light upon light producing darkness can be readily understood by considering the analogous case of ripples on the surface of water. Let a continuous series of ripples be produced by a vibrating metal strip dipping in and out of the surface of water. A light floating body, say a leaflet, wobbles up and down with the same frequency as the vibrator. The ripples spread out in widening circles. If now another vibrator of the same frequency is brought near the first, the appearance of the ripples is completely changed. Along certain radial lines starting from the two vibrators, the water surface seems undisturbed; the leaflet does not move if placed along these lines. In between these lines the ripples have a very large amplitude.

At point equidistant from the two vibrators, that is, along the perpendicular bisector of the line joining the vibrators, the waves from both arrive in phase. Both systems of waves tend to move the water up or down at the same time. The amplitude of the waves along this line is double that due to either of the systems of waves. At some other point sufficiently away from the perpendicular bisector, the crest of one system arrives at the same time as the trough of another, and thus the two systems cancel each other. Destructive interference occurs if the distances of the two vibrators

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from the point differ by $(n + \frac{1}{2})\lambda$, where λ is the wavelength and n is zero or an integer. Constructive interference occurs if the path difference is $n\lambda$.

An important condition for interference is that the two systems of waves are coherent, that is, that they have always the same phase relation to each other. If the two slits P and Q in Young's experiment were illumined by waves from two different sources, interference effects would not be observed. The reason is that waves produced by two sources would have no phase relation to each other. When the two slits are illumined by different parts of the same wave-front, at any point beyond the slit they always arrive with a constant difference in phase.

THEORY

The mathematical expression for a progressive wave is

$$S = a \cos 2\pi\nu\left\{\left(t-\frac{x}{v}\right) + \alpha\right\}, \tag{1}$$

where S is the magnitude of the electric or magnetic field, also called displacement, at time t and distance x, a is the amplitude, ν the frequency, v the velocity, and α a term denoting the phase. Due to the superposition of two waves, denoted by subscripts 1 and 2,

$$\begin{aligned} S = S_1 + S_2 &= a_1 \cos 2\pi\nu\left\{\left(t-\frac{x}{v}\right) + \alpha_1\right\} \\ &+ a_2 \cos 2\pi\nu\left\{\left(t-\frac{x}{v}\right) + \alpha_2\right\}. \end{aligned} \tag{2}$$

Both waves have the same frequency and velocity, but different amplitudes and phases. x is measured from any arbitrary point.

The displacement at x=0 is given by

$$S = A \cos(2\pi\nu t + \alpha), \tag{3}$$

where A is the amplitude and α the phase of the resulting wave. Expanding the right hand sides of equation (2) with x=0 and equation (3), equating coefficients of $\cos 2\pi\nu t$ and $\sin 2\pi\nu t$, squaring and adding, it is seen that

$$A^2 = a^2 + a_2{}^2 + 2a_1a_2 \cos(\alpha_1 - \alpha_2). \tag{4}$$

As $\cos(\alpha_1 - \alpha_2)$ varies between +1 and -1, A varies between $(a_1 + a_2)$ and $(a_1 - a_2)$. If the amplitudes a_1 and a_2 are both equal to a, the minimum value of A is 0 and the maximum value is 2a. Since the intensity is proportional to the square of the amplitude, the intensity at the maximum is 4 times that due to either wave. The condition for maximum brightness is: $\alpha_1 - \alpha_2 = 2n\pi$, which corresponds to a path difference of $n\lambda$.

Let R and S, figure 1, be the positions of the central fringe and the nth bright fringe below R. Let PQ=s, RS=x, and let D be the distance of the screen from the two slits.

$$PS^2 - QS^2 = \left\{D^2 + (x + \tfrac{s}{2})^2\right\} - \left\{D^2 + (x - \tfrac{s}{2})^2\right\}$$

$$= 2xs$$

$$PS - QS = 2xs/(PS + QS) = 2xs/2D,$$

since PS + QS is very nearly equal to 2D.

$$PS - QS = n\lambda \text{ , so that } \lambda = \frac{1}{n}\frac{xs}{D}. \qquad (5)$$

Equation (5) formed the basis for the first experimental determination of the wavelength of light.

Young's experiment in its original form is difficult to perform and failed to carry conviction when the result were first published. If $s = 1$ mm, $D = 2$ meters, the distance between successive fringes for sodium yellow light ($\lambda = 5.89 \times 10^{-5}$ cm) is only 1.2 mm. The illumination is too poor; the fringes are too close; and two fine slits at 1 mm distance are difficult to produce. The controversy as to whether light is propagated as corpuscles or waves had existed for over a century and a half. Francesco M. Grimaldi, who is regarded as the founder of the wave theory of light, in his book, _Physico-Mathesis de Lumine, Coloribus et Iride_, published in 1665, describes several experiments on diffraction and interference of light, and presents the rudiments of a wave theory. Newton discusses several diffraction effects in his _Opticks_, published in 1708; he threw his weight heavily on the side of the corpuscular theory of light. Experiments more convincing than those of Young were needed to overthrow a theory based on Newton's authority. Between 1814 and 1816, Fresnel introduced two better methods of producing interference fringes, he also gave a more complete theory of the formation of the fringes, based on the hypothesis of secondary wavelets which was first developed in 1678 by Huygens. Huygens' hypothesis was that every point on a wave-front acted as the source for a secondary train of waves, and that the envelope of these secondary waves determined every successive position of the wave-front.

The two improved experimental arrangements introduced by Fresnel were the bimirror and the biprism. These solve the difficulty of obtaining two slits sufficiently narrow and close to each other. In the bimirror arrangement, light from a narrow slit is reflected by two plane mirrors inclined at a small angle to each other. Thus the two slits are replaced by the two images of a single slit, and the distance between these can be adjusted by changing the angle between the mirrors. In the biprism arrangement two small angle prisms joined at their base each produce a small deflection of the light emerging from a single slit, and thus cause two sets of coherent waves to be superposed. A single mirror may also be used as devised by Lloyd to produce interference between wave trains produced by a slit and its image.

APPLICATIONS OF INTERFERENCE EFFECTS

There are many interesting applications of the principle of interference. Refractometers based on interference effects are used to measure small changes in the refractive index of transparent media. In the Rayleigh refractometer (see figure 2) light from a linear source S made parallel by a lens L_1 is split into two beams by two fairly wide slits, and then made to pass through two similar tubes T_1 and T_2. After transmission through the tubes the two beams are brought to a common focus by another lens L_2. If the two tubes contain transparent media of the same refractive index, say the same liquid, the center of the fringe pattern is formed on the axis of the instrument. If the refractive index of the liquid in one of the two tubes is changed, as for example by introducing a solvent, the fringes shift across the focal plane of the viewing lens. By counting the number of fringes which cross a reference line, the equivalent path difference and hence the change in refractive index can be calculated. Compensator plates M_1, M_2, which restore the fringe pattern to its original position are a convenient device for counting the fringe shift.

Michelson's method of measuring stellar diameters is another application of interference. A beam mounted over the entrance aperture of a large telescope carries four mirrors as shown in figure 3. The arrangement is similar to that of Young's experiment, with the mirrors M_1,M_2 as the slits and the star as the source. The diffraction image of the star is crossed by interference bands if M_1 and M_2 are relatively close to each other. As the distance between them is increased, the fringes become less distinct and finally disappear. This is due to the fringe pattern due to one half of the star being completely cancelled by that due to the other. If the distance between M_1 and M_2 for disappearance of the fringes is s, r the distance of the star, d its diameter, and λ the wavelength of light, $\frac{d}{r} = 1.22 \frac{\lambda}{s}$. The first star to be measured by this method was Betelgeuse, for which the bands disappeared at s = 306.5 cm. Substituting values 5.75×10^{-5} cm for λ, and 1.712×10^{20} cm for r, as determined from the parallax of Betelgeuse, the diameter of the star is found to be 3.918×10^{13} cm, which is 31 percent greater than the diameter of the earth's orbit round the sun. Several other near stars of large size have since been measured by the same method.

Thin films of transparent media produce striking interference effects, as for example, when a few drops of gasolene are spilled over a wet pavement. The two wavetrains which interfere in this case are those reflected from the upper surface of the oil film and from the water-oil interface. The colors of butterfly's wings and sea shells have a similar origin. The so-called Newton's rings are produced by the air film between two partially reflecting, spherical surfaces. The optical quality of a glass surface can be tested by causing interference fringes between it and a standard test plate of high optical quality. The fringes are analogous to contour lines in geographical maps, each new fringe indicating deviation from true flatness by half a wavelength.

INTERFEROMETERS

Interferometers are instruments of high precision measurement based on the principle of interference. A beam of light is divided into two or more beams by partial reflection and transmission, and are recombined after they have travelled different path distances. Of the many different types of interferometers only two which are widely used will be described here, the Michelson interferometer and the Fabry-Perot interferometer.

The Michelson interferometer is shown schematically in figure 4. Monochromatic light from an extended source is collimated by a lens L_1 and falls on the beam splitter M of which the hind surface partially reflects half the intensity upwards and transmits the other half. The two halves of the beam are returned by the mirrors M_1 and M_2, and the interference pattern is viewed in the focal plane of the lens L_2. C is a compensator plate similar to M, which gives to the beam from M_2 an extra path equal to that travelled by the other beam in the beam splitter. The form of the fringes depends on the adjustment of the two mirrors M_1 and M_2. If they are not quite at right angles to each other, and the difference in path of the two beams is small, the image of M_2 in M forms a thin wedge with the front surface of M_1. The fringes are straight and parallel to the apex of the wedge. If the difference in path is large the two mirrors should be adjusted so that the image of M_2 in M is exactly parallel to M_1. In this case the fringes are circular. Each circle is due to pencils of light which have a constant inclination to the axis of the lens L_1.

Two of the applications of the Michelson interferometer are of historic importance: the standardization of the meter in terms of the wavelength of light and the Michelson-Morley experiment for the drift of ether. If one of the two mirrors is moved parallel to itself the pattern of fringes shifts across the field of view. The displacement of the mirror for each fringe shift is $\lambda/2$. This method was first used by Michelson and Benoit in 1892 for comparing the red line (6438 Å) of cadmium with the International Prototype Meter which is kept in Paris, France. More precise measurements of the meter in terms of the wavelength of light have since been made by other observers. Since the wavelength of light is a more reliable standard and can be measured with greater accuracy, it was judged desirable to replace the standard meter by a suitable spectral line as the standard of length. The International Commission of Weights and Measures formally adopted in 1960 the orange red line of krypton 86 as the stnadard and defined the meter as exactly 1 650 763.73 wavelengths in vacuum of this line.

The Michelson Morley experiment of 1887 was an attempt to measure the speed of the ether "wind" past the moving earth. If one arm of the interferometer is in the direction of the earth's motion relative to the ether and the other at right angles to this motion, the relative path difference between the two beams of light is nearly L v^2/c^2, where L is the length of each arm, v is the velocity of the earth and c is the velocity of light. By floating the interferometer in a pool of mercury and rotating it through 90°, a fringe shift corresponding to twice this path difference should be observed. Accurate experiments hsowed that the fringes did not shift. This negative result served as the basis for the theory of relativity.

A recent application of the Michelson interferometer is for spectrophotometry of composite sources. The method is especially applicable for the infrared range. One of the two mirrors is moved parallel to itself at a very constant rate. The variation of intensity over a small area at the center of the ring system at P is measured by an infrared detector. If the source were strictly monochromatic the output signal of the detector would vary sinusoidally with time; and the displacement of the mirror between successive maxima is half a wavelength. With a composite source as input, the output is the sum of a large number of sine functions, each of them being due to the energy in a narrow wavelength band of the source. A Fourier transform of the output signal, as may well be obtained with the aid of a digital computer, gives the spectral energy distribution of the source. The compactness of the instrument is a special advantage compared to infrared prism monochromators, and hence several designs of the interferometric spectrophotometer have been developed for use in satellites and space probes.

The Fabry-Perot interferometer consists of two parallel plates of glass or quartz. The inner surfaces are optically flat and semi-silvered. Light from an extended source is made parallel by a lens L_1, passes through the interferometer and is focussed by another lens L_2. A system of circular fringes is observed in the focal plane of L_2. The path of an oblique ray of light in between the two plates is shown in figure 5. Interference takes place between the directly transmitted ray and the rays that undergo one or more reflections between the plates. A high degree of wavelength resolution is the main advantage of the Fabry-Perot interferometer. In a typical case of 1 cm separation of plates and 90 percent reflectance, wavelength resolution is over a million; that is, two wavelengths 0.005 Å apart at 5000 Å will give completely distinguishable ring systems.

By increasing the reflectance of the plates or the separation between them the wavelength resolution can be increased to any desired degree. Laser beams which give highly coherent single wavelengths permit plate separation of over a meter. Extensive use has been made of the Fabry-Perot interferometer for precision measurement of wavelengths of spectral lines.

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Suggestions for cross references

Electromagnetic field
Wave theory
Isaac Newton
Refractive index
Relativity

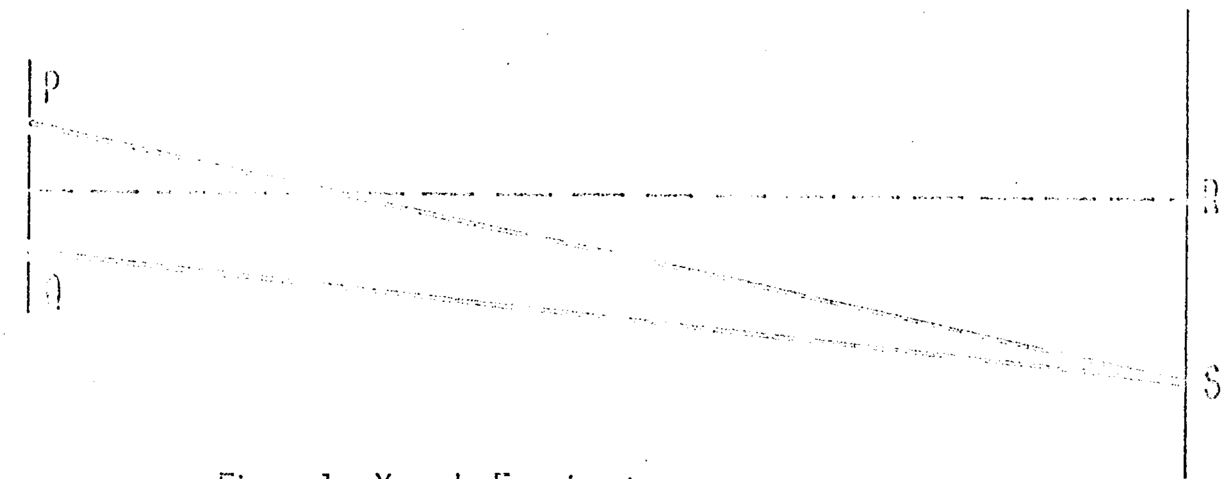

Figure 1. Young's Experiment

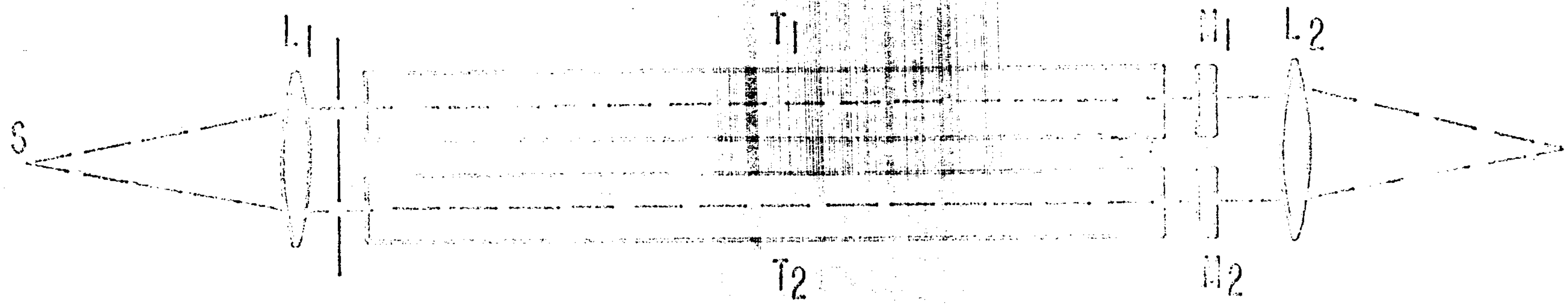

Figure 2. Rayleigh Refractometer

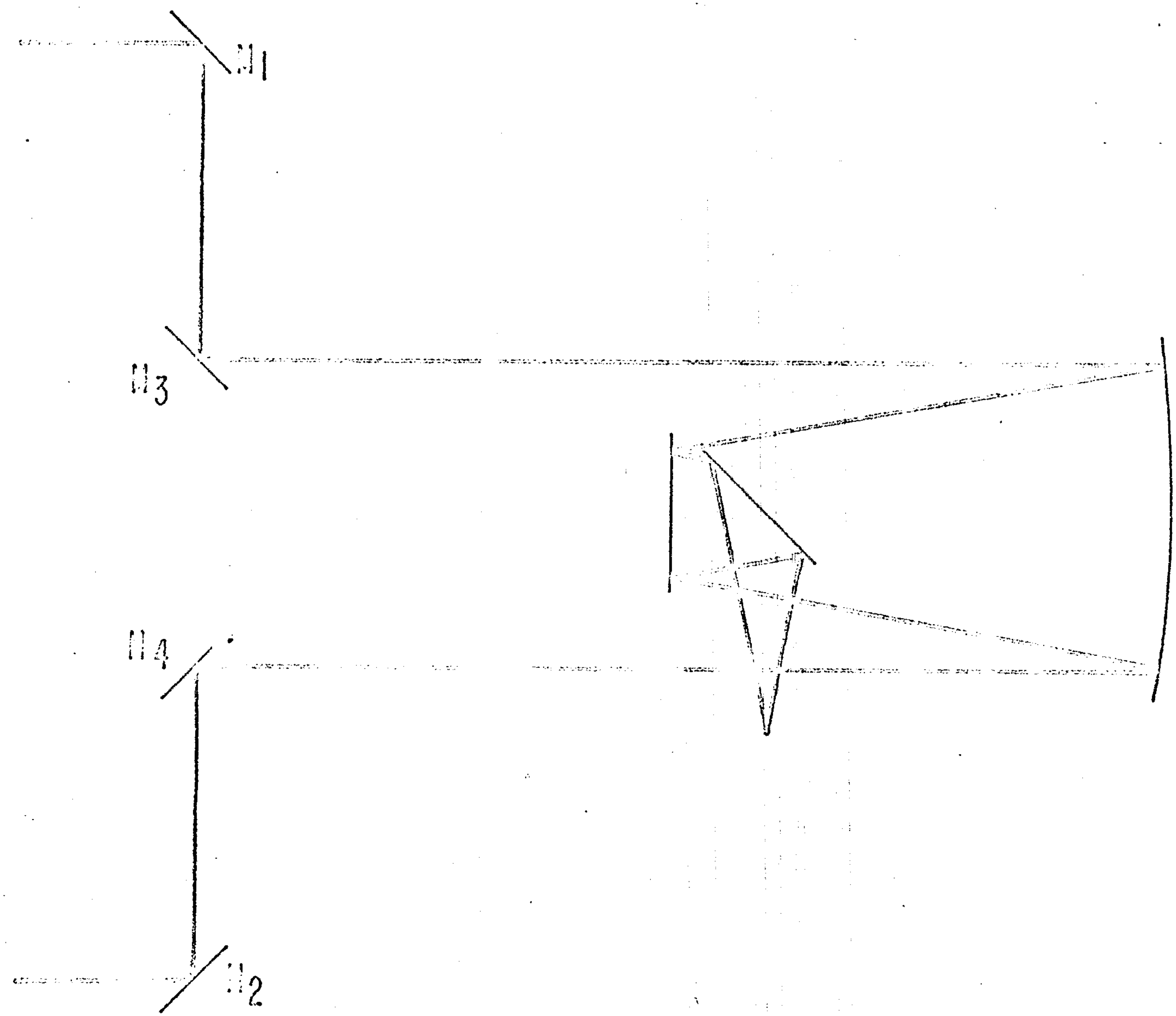

Figure 3. Stellar Interferometer

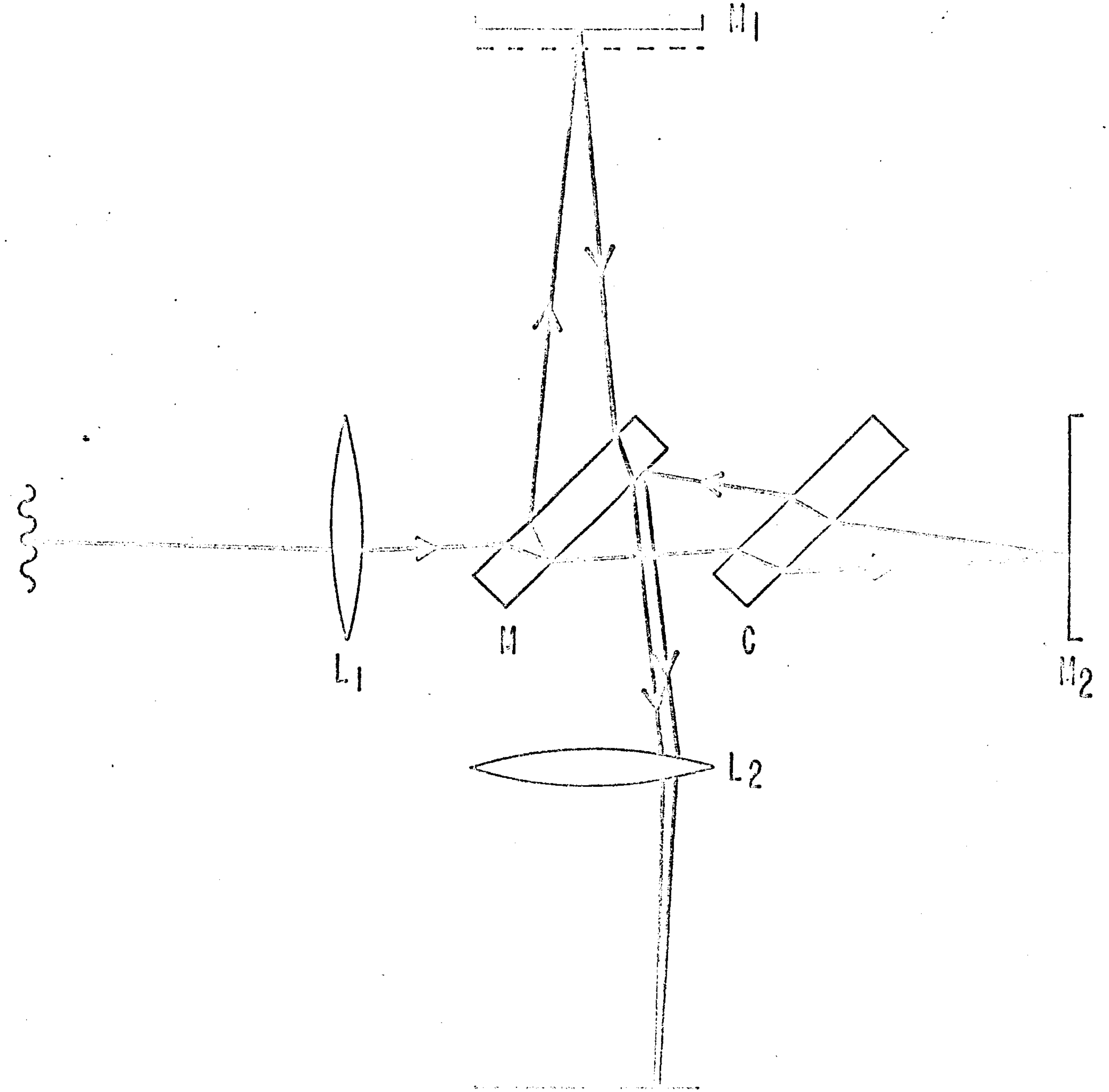

Figure 4. Michelson Interferometer

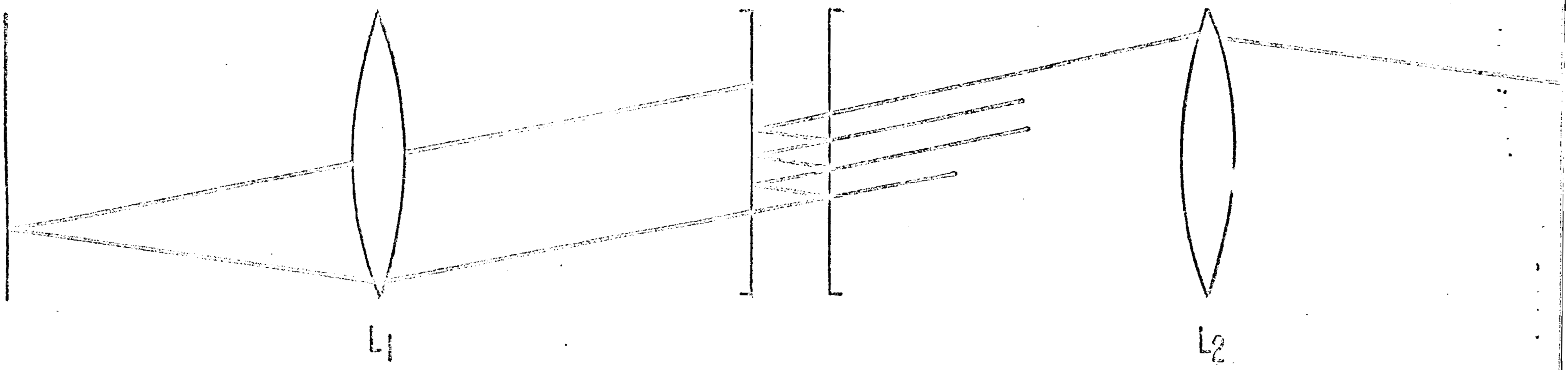

Figure 5. Fabry-Perot Interferometer

This paper was written at the request of Dr. Robert M. Besancon, Editor, *Encyclopedia of Physics*. The Encyclopedia is due to be published by Reinhold Publishing Company, New York. There is no call for an article of this type being published as a NASA document.